A Novel Retinex Model Based on Sparse Source Separation

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ABSTRACT
Retinex was introduced by E.Land to explain and solve a problem of color constancy in human visual system (HVS). In this paper, we propose a novel Retinex model based on sparse source separation problem. Different from the existing models, we can explain a relation between the modeling and the effectiveness of Retinex decomposition with the proposed model. We demonstrate the performance of our model by experimental results.

Keywords
color constancy, Retinex theory, modeling, gradient domain, sparse source separation

1. INTRODUCTION
The color of object which is determined by machine visual system (MVS), such as a digital camera, is based on the amount of reflected light on the object. However, when human visual system (HVS) determines the object color, it also considers the amount of detail in the surrounding area and the variation of overall illumination. With this complex system, we can automatically discount the variation of illumination and so easily recognize the color of object which is same under varying illumination conditions [Pal09][Riz07]. This feature of the HVS is called color constancy [Ebn07], and it has been studied over the forty years.

Land's Retinex theory is the first computational model to simulate and explain the color constancy of HVS [Lan71]. He simulated and explained how the HVS perceives color, based on experiments using Mondrian patterns. With this result, he proposed path-based Retinex algorithm to solve the discrepancy problem between the MVS and HVS [Lan83]. This algorithm extracts the reflectance components from the original image which is obtained by MVS. The extracted reflectance image is so clear and quite similar to the image of HVS because it has no illumination components in the image. After Land's pioneering studies of Retinex theory, there have been many researches to interpret, improve, and implement the Retinex algorithm.

The Retinex algorithm is usually categorized into four areas: path-based algorithms, recursive algorithms, center/surround algorithms and physics-based algorithms[Mor09] . Among these areas, the physics-based algorithms are widely studied in recent years because they can efficiently remove the global illumination from the images. Based on the main assumptions that the illumination varies smoothly and the reflectance is piecewise constant, the physics-based Retinex algorithms set models of the reflectance and the illumination firstly, and then decompose image intensity as a product of the reflectance and the illumination.

In [Kim03], Kimmel et al. proposed a variation model for Retinex which set the illumination as the variational framework. This algorithm can simply extract the illumination using the steepest descent method, but the reflectance model is not considered. To improve this algorithm, Michael et al. proposed a total-variation model for Retinex [Mic11]. This algorithm uses the total-variation model for the reflectance[MaW10], and also considers the illumination model using the assumption of spatially smoothness. This algorithm makes the decomposition of image more appropriate and reasonable than the previous works. However, there still remains a problem in the total-variation model that the mathematical relation between the modeling and the effectiveness of decomposition is not clear.

In this paper, we propose a sparse source separation model of Retinex theory. Using the properties that the gradient of reflectance is spatially sparse and the gradient of illumination is sparse in frequency domain, we apply the sparse source separation algorithm to the Retinex model. With our model, we can explain the effectiveness of Retinex decomposition and can decompose the reflectance and illumination more accurately. Some experimental results are presented to show the effectiveness of the proposed model.

2. VARIATION/TOTAL-VARIATION MODEL FOR RETINEX

As mentioned above, the variation and total-variation model for Retinex are categorized in the physics-based Retinex algorithm. The algorithms in this category decompose the image intensity S as a product of the reflectance R and of the incident illumination intensity L as follows [Mor09] :

$$S = R \cdot L \tag{1}$$

,where $0 < R < 1$ and $0 < L < \infty$. In order to handle the product form, they are converted into the logarithmic domain, i.e.,

$$s = r + l \tag{2}$$

where $s = \log S$, $r = \log R$, and $l = \log L$. Based on the assumption that the illumination is spatially smooth, Kimmel et al. proposed the variation modeling for Retinex [Kim03], i.e.,

$$\begin{aligned} \underset{l}{\operatorname{argmin}} F[l] = \int_{\Omega} & (|\nabla l|^2 + \alpha(l-s)^2 \\ & + \beta|\nabla(l-s)|^2)dxdy \end{aligned}$$

$$\text{subject to :}\ l \geq s, \text{and } \langle \nabla l, \vec{n} \rangle = 0 \text{ on } \partial\Omega \tag{3}$$

where Ω is the support of the image, $\partial\Omega$ its boundary, and $\vec{n}$ is the normal to the boundary. α and β are free non-negative real parameters. This model is a quadratic programming problem which can be solved by many methods such project normalized steepest descent method as in [Kim03]. However, this model has some limitations because the reflectance piecewise constant assumption is not considered in its model.

To improve the variation model of Retinex, Michael et al. proposed the total-variation model for Retinex [Mic11], i.e.,

$$\begin{aligned} \underset{r,l}{\operatorname{argmin}} E(r,l) = \int_{\Omega} |\nabla r| + \frac{\alpha}{2}\int_{\Omega} |\nabla l|^2\, dx \\ + \frac{\beta}{2}\int_{\Omega} (l + r - s)^2 dx \\ + \frac{\mu}{2}\int_{\Omega} l^2 dx \end{aligned} \tag{4}$$

where α, β, and μ are positive numbers for regularization parameters, the term $\int_{\Omega} (l + r - s)^2 dx$ is used for the fidelity, and the term $\int_{\Omega} l^2 dx$ is used only for the theoretical setting. As shown in (4), they considered the both sides: the reflection function is piecewise constant, and the illumination function is spatially smooth. Using these properties, the total-variation model makes more appropriate and reasonable for the decomposition of Retinex than the previous variation model.

Nevertheless, there still remain some problems in the variation/total-variation model. The variation and total-variation model do not explain the mathematical relation between their modeling and the possibility of decomposition. So we do not know whether the modeling is suitable or not for the Retinex decomposition. They also have some halo effects in their decomposed reflectance images because they consider both the reflectance and illumination in the spatial domain. Due to the fact that the illumination was modeled as smooth in the spatial domain, the results of reflectance images lead to the creation of local halos, such as those around the letters.

3. PROPOSED RETINEX MODEL

In order to improve the previous model of Retinex, we present a sparse source separation model of Retinex. In this section, we introduce the sparse source separation algorithm firstly, and then we present a novel Retinex decomposition model using the sparse source separation algorithm.

Source separation algorithm in mixed two sparse signals

Suppose that there is a mixed signal $z \in \mathbb{R}^d$ of the superposition model, i.e.,

$$z = x + y \tag{5}$$

We assume that a vector $x \in \mathbb{R}^d$ is sparse if the most of its entries are equal to zero. Similarly, a vector $y \in \mathbb{R}^d$ is sparse in frequency if its discrete cosine transform (DCT) Dy is sparse, where $D \in \mathbb{R}^{d \times d}$ is the matrix that encodes the DCT. Using these properties, we can separate the z into x and y to search for the sparsest possible components [McC14], i.e.,

$$[\check{x}, \check{y}] = \underset{x,y \in \mathbb{R}^d}{\operatorname{argmin}}\{\|x\|_0 + \lambda\|Dy\|_0 : z = x + y\} \tag{6}$$

Where the ℓ_0-norm measures the sparsity of its input, and $\lambda > 0$ is a regularization parameter that trades the relative sparsity of solutions.

In [Don01], Donoho et al. proved that if

$$\|x\|_0 + \|Dy\|_0 \leq \sqrt{d/2}, \tag{7}$$

then the solution to equation (6) is unique. In other words, we can perfectly separate x and y from z if x and Dy are sufficiently sparse.

Unfortunately, solving (6) involves an intractable computation problem. So we replace the ℓ_0 penalty with the convex ℓ_1-norm to solve a classical sparse approximation program as follows :

$$[\hat{x}, \hat{y}] = \underset{x,y \in \mathbb{R}^d}{\operatorname{argmin}} \{\|x\|_1 + \lambda \|Dy\|_1 : z = x + y\} \quad (8)$$

In [Don01], Donoho et al. also proved that if

$$\|x\|_0 + \|Dy\|_0 \leq \frac{1}{2}\sqrt{d/2}, \quad (9)$$

then the solution to equation (8) is also unique. In other words, we can also perfectly separate x_0 and y_0 from z using the ℓ_1-norm if x and Dy are sufficiently sparse.

The change to the convex ℓ_1-norm offers a benefit that we can use a number of highly efficient convex program algorithms for solving (8).

A sparse source separation model of Retinex theory

At first, we convert the equation (2) into a gradient domain, i.e.,

$$\nabla s = \nabla r + \nabla l \quad (10)$$

, because the reflectance r is piecewise constant, the gradient of reflectance ∇r is spatially sparse. And because the illumination l is spatially smooth, the gradient of illumination ∇l is piecewise constant in spatial domain, so it is sparse in frequency domain.

As we explained in the previous subsection, we can efficiently decompose the superposition signal which is mixed the spatially sparse signal and the sparse-in-frequency signal. So, applying this fact to the sparse source separation problem, we can decompose the ∇s into ∇r and ∇l, solving the follow problem :

$$[\check{r}', \check{l}'] = \underset{r', l' \in \mathbb{R}^N}{\operatorname{argmin}} \{\|r'\|_0 + \lambda \|Dl'\|_0 : s' = r' + l'\} \quad (11)$$

where $r' = \nabla r, l' = \nabla l, s' = \nabla s,$ and N is the size of image. Using the decomposed r' and l', we can get the reflectance r and illumination l with the simple inverse process of equation (10).

Based on the equation (7), we can perfectly decompose s' into r' and l' if

$$\|r'\|_0 + \|Dl'\|_0 \leq \sqrt{N/2} \quad (12)$$

So, if r' and l' satisfy the above sparse condition, then we can accurately decompose the reflectance image from the original image.

However, as we mentioned before, it is difficult to solve the equation (11) because the ℓ_0 optimization problem is NP-hard. So we replace the ℓ_0 penalty of (11) with the ℓ_1-norm, i.e.,

$$[\hat{r}', \hat{l}'] = \underset{r', l' \in \mathbb{R}^N}{\operatorname{argmin}} \{\|r'\|_1 + \lambda \|Dl'\|_1 : s' = r' + l'\} \quad (13)$$

Based on the equation (9), we also see that the equation (13) is perfectly decomposed if

$$\|r'\|_0 + \|Dl'\|_0 \leq \frac{1}{2}\sqrt{N/2} \quad (14)$$

With our Retinex modeling, we can explain the relation between the modeling and the efficieny of Retinex decomposition that the sparsity of ∇r and $D\nabla l$ are proportional to the accuracy of Retinex decomposition, and if they satisfy the equation (14), then we can perfectly decompose the reflectance and illumination from the original image.

To solve the equation (13), we change this equation to the unconstrained form, i.e.,

$$[\hat{r}', \hat{l}'] = \underset{r', l' \in \mathbb{R}^N}{\operatorname{argmin}} \left\{ \|r'\|_1 + \lambda \|Dl'\|_1 + \frac{\beta}{2} \int (s' - r' - l')^2 dx \right\} \quad (15)$$

,where x is the axis of image, and β is the positive regularization parameter. To solve the equation (15), we use the alternating minimization scheme as

$$r'^{(k+1)} = \underset{r' \in \mathbb{R}^N}{\operatorname{argmin}} \left\{ \|r'\|_1 + \frac{\beta}{2} \int \left(s' - r' - l'^{(k)}\right)^2 dx \right\}$$

$$l'^{(k+1)} = \underset{l' \in \mathbb{R}^N}{\operatorname{argmin}} \left\{ \lambda \|Dl'\|_1 + \frac{\beta}{2} \int \left(s' - r'^{(k+1)} - l'\right)^2 dx \right\} (16)$$

In order to get the r and l directly without the inverse process of equation (10), we apply the split Bregman method [Gol09] in each subproblems in (16), i.e.,

$$r^{(k+1)} = \underset{r, d_1 \in \mathbb{R}^N}{\operatorname{argmin}} \left\{ \|d_1\|_1 + \frac{\beta}{2} \int \left(\nabla s - \nabla r - \nabla l^{(k)}\right)^2 dx \right\}$$

$$l^{(k+1)} = \underset{l, d_2 \in \mathbb{R}^N}{\operatorname{argmin}} \left\{ \lambda \|d_2\|_1 + \frac{\beta}{2} \int \left(\nabla s - \nabla r^{(k+1)} - \nabla l\right)^2 dx \right\} (17)$$

,where $d_1 = \nabla r$ and $d_2 = D\nabla l$. With the iteration of each sub-problems, we can get the decomposed reflectance and illumination image.

4. EXPERIMENTAL RESULTS

In our implementation, we use the HSV Retinex which considers the intensity layer (V) only. We compare the results of our model with those of Kimmel's model [Kim03] and the Michael's model [Mic11]. The parameters of previous models set the same as their papers. In our model, we fix $\lambda = 4$ and $\beta = 0.1$ which apply to weight the illumination cost.

Figure 1 and 2 show the experimental results of the previous works and our proposed model with two Retinex test images. In these figures, we can see that the reflectance image of our model is the most clear in the edge of image.

In the next experiments, we apply the Retinex algorithms using the color circle images we made. As shown in figure 3, our proposed model restores the edge and intensity of reflectance more accurately than the other Retinex algorithms. Figure 4 shows the differences of S-CIELAB image [Zha97] between the

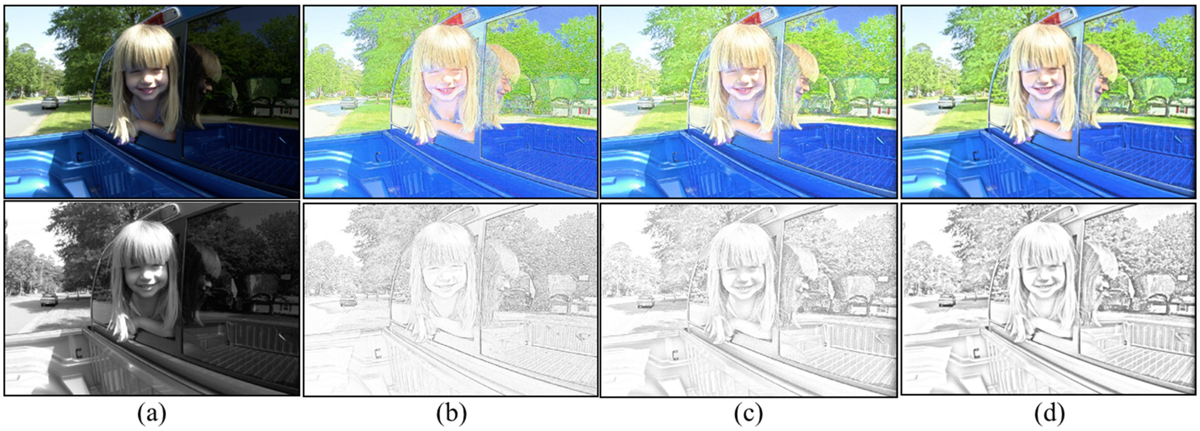

Fig. 1 The results of Retinex algorithms with 'girl1' test image. Top is the reconstructed color image, and bottom is the mono reflectance image of V layer. (a) The original image; (b) reflectance image by the variation model in [Kim03]; (c) reflectance image by the total-variation model in [Mic11]; (d) reflectance image by the proposed model.

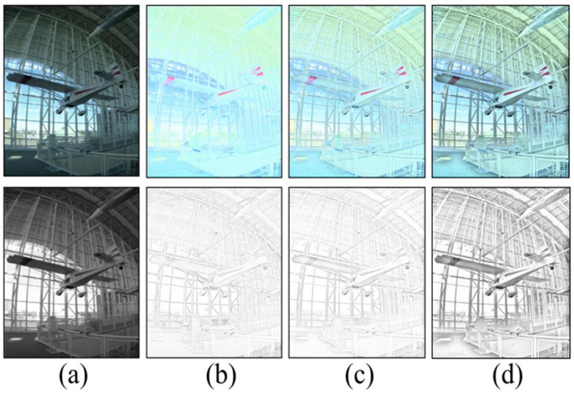

Fig 2 The results of Retinex algorithms with 'interior1' test image. Top is the reconstructed color image, and bottom is the mono reflectance image of V layer. (a) The original image; (b) reflectance image by the variation model in [Kim03]; (c) reflectance image by the total-variation model in [Mic11]; (d) reflectance image by the proposed model.

original color circle and the reflectance image which is obtained with Retinex algorithm. The brightness of S-CIELAB image means the amount of errors. As shown in figure 4, our Retinex model shows the less S-CIELAB differences than the other Retinex algorithms.

With the results in figure 5, we can also see that our model is almost free for the local halo effect, yet the results of previous model in figure 5(b)(c) lead to the creation of local halos.

With these results, we can say that our model is effective for the Retinex decomposition.

5. CONCLUSIONS

In this paper, we propose a sparse source separation model of Retinex algorithm. Based on the sparse source separation problem, we use the properties of illumination and reflectance in the source separation model that the gradient of illumination is sparse-in-frequency and the gradient of reflectance is sparse in spatial domain. Unlike the previous physics-based Retinex models, our model is able to explain the modeling and the efficiency of model for Retinex decomposition. Some experimental results show that our Retinex model is effective to decompose the reflectance components from the original images.

6. ACKNOWLEDGMENTS

"This research was supported by the MSIP(Ministry of Science, ICT and Future Planning), Korea, under the ITRC(Information Technology Research Center) support program (IITP-2015-H8501-15-1001) supervised by the IITP(Institute for Information & communications Technology Promotion)"

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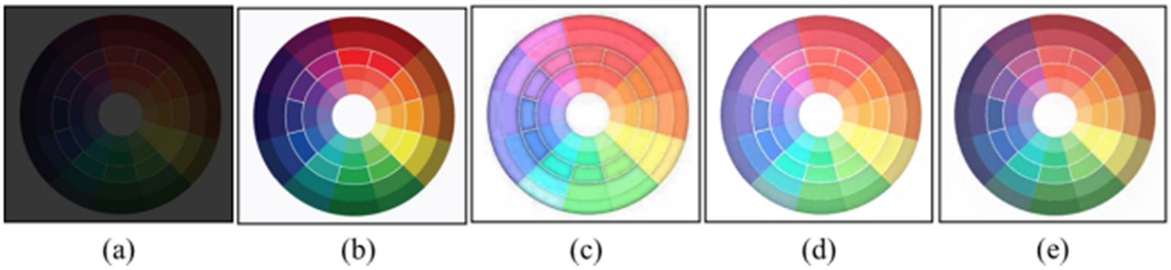

Fig 3 The results of Retinex algorithms with ‘color circle’ test image. (a) The illuminated image; (b) The original image; (c) reflectance image by the variation model in [Kim03]; (d) reflectance image by the total-variation model in [Mic11]; (e) reflectance image by the proposed model.

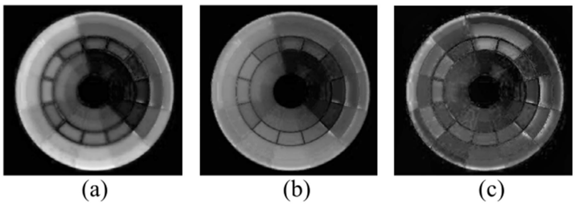

Fig 4 The S-CIELAB difference images between the original color circle image and the results of Retinex reflectance images. (a) With the reflectance image by the variation model in [Kim03]; (b) reflectance image by the total-variation model in [Mic11]; (e) reflectance image by the proposed model.

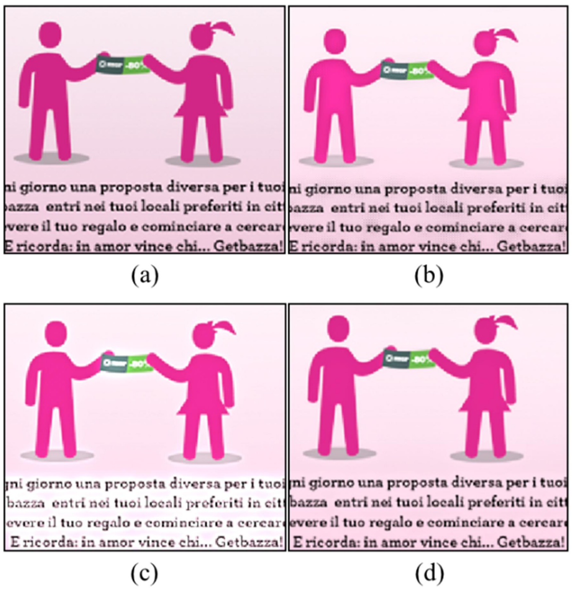

Fig 5 The results of Retinex algorithms with ‘text’ test image. Top is the reconstructed color image, and bottom is the mono reflectance image of V layer. (a) The original image; (b) reflectance image by the variation model in [Kim03]; (c) reflectance image by the total-variation model in [Mic11]; (d) reflectance image by the proposed model.

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 ISBN 978-80-86943-67-1